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The geometry of Grassmannian manifolds and Bernstein-type theorems for higher codimension

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Abstract. This paper continues the line of research where one shows a Bernstein-type theorem, namely, that a complete minimal submanifold of a Euclidean space has to be affine linear, by proving that its Gauss map is constant. This Gauss map is a harmonic map into some Grassmann manifold. Therefore, we need geometric conditions on the target of the Gauss map which imply that, being harmonic, it has to be constant. For this purpose, we identify a region $\mathbb{W}_{\frac{1}{3}}$ in a Grassmann manifold $\mathbf{G}_{n,m}$, not covered by a usual matrix coordinate chart, with the following important property. For a complete $n-$submanifold M of $\mathbb{R}^{n+m}$ with $n \geq 3$, and $m \geq 2$, with parallel mean curvature whose image under the Gauss map is contained in a compact subset $K \subset \mathbb{W}_{\frac{1}{3}} \subset \mathbf{G}_{n,m}$, we can construct strongly subharmonic functions on M and derive *a priori* estimates for the harmonic Gauss map, to eventually show that it is constant. While we do not know yet how close our region is to being optimal in this respect, it is substantially larger than what could be achieved previously with other methods. Consequently, this enables us to obtain substantially stronger Bernstein-type theorems in higher codimension than previously known. We also provide some new explicit computations for the example of Lawson and Osserman [18].

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